A SIMPLE INDUCTION PROOF ABOUT A SEQUENCE

The sequence (d_n) is defined as follows:

$d_1 = 3$ and, for every integer $t \geq 2$, $d_t = 2d_{t-1} + 1$.

<u>To Prove</u>: For every integer $n \geq 1$, $d_n = 2^{(n+1)} - 1$.

Proof:

[Basis Step] Let $n = 1$. $d_n = d_1 = 3$, by the sequence definition.

$2^{(n+1)} - 1 = 2^{(1+1)} - 1 = 2^2 - 1 = 4 - 1 = 3$

$\therefore$ For $n = 1$, $d_n = 2^{(n+1)} - 1$, by substitution.

[INDUCTIVE STEP]

Let k be any integer such that $k \geq 1$.

Suppose $d_k = 2^{(k+1)} - 1$. [Inductive Hypothesis]

[NTS: $d_{k+1} = 2^{(k+1)+1} - 1$.]

[We need to verify the formula $d_t = 2d_{t-1} + 1$ applies to d_{k+1}.]

Since $k \geq 1$, $k+1 \geq 2$.

Since $k+1 \geq 2$, $\underline{d_{k+1} = 2d_k + 1}$, by the sequence def'n.

Recall that $d_k = 2^{(k+1)} - 1$, by the Inductive Hypothesis.

$\therefore d_{k+1} = 2[2^{(k+1)} - 1] + 1$, by substitution,

$= 2 \cdot 2^{(k+1)} - 2 + 1 = 2^{(k+2)} - 1$,

$\therefore d_{k+1} = 2^{(k+1)+1} - 1$. [Which is what we needed to show.]

$\therefore$ FOR ALL INTEGERS $k \geq 1$, if $d_k = 2^{(k+1)} - 1$, Then $d_{k+1} = 2^{(k+1)+1} - 1$, by Direct Proof.

[END of INDUCTIVE STEP]

$\therefore$ By Mathematical Induction, $d_n = 2^{(n+1)} - 1$, for all integers $n \geq 1$. QED